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diate neighbourhood of the place. On trying to break the large stone, or move it with crowbars, it was ascertained that it was not very thick; and with the assistance of a large sledge hammer it was broken into several pieces. One of these fell down, leaving an opening in the roof of a chamber or tomb. The stone now broken appeared to have been originally placed on others, which formed the sides of a complete kiswain, very similar to that described in the Proceedings of the Academy (vol. i. p. 188). Like that one found in the Phœnix Park, it contained a skeleton, whose head has all the characteristics which distinguish the two found in that tomb; but in this, the vase or urn, herewith presented, was found within the limits of the chamber, and placed on the north side of the skeleton. It was about half full of a black sooty substance, but it contained no bones like the urns found in the Park. Its contents were examined by the people present, and, not being supposed to be of any interest, were thrown away.

Near the tomb were discovered a number of small chamber tombs, without covering stones. These had all been previously opened. Fragments of burned bones were discovered in several; and on the east of the kiswain was found a pit, about five feet deep, with walled sides. This appeared to have been used as a depository for burned bones and ashes, with which it was filled. At some distance the fragment of the urn also presented was found near the surface. The character or style of the workmanship differs from that of the urn found in the tomb, but it exactly resembles an urn in the Museum, found, under similar circumstances, at the hill of Rath, near Drogheda.

Professor Graves communicated the following note:

It has been shown by Professor MacCullagh* that the equation of the central surface of the second order,

$$\frac{x^2}{a_0^2} + \frac{y^2}{b_0^2} + \frac{z^2}{c_0^2} = 1,$$

* Proceedings of the Royal Irish Academy, vol. iii. p. 429.

may be reduced by a transformation of coordinates to the form

$$\frac{\xi^2}{k^2} + \frac{\eta^2}{k'^2} + \frac{\zeta^2}{k''^2} = (f-1)\left\{\frac{\xi_0\xi}{k^2} + \frac{\eta_0\eta}{k'^2} + \frac{\zeta_0\zeta}{k''^2} - 1\right\}^2$$

The new origin being fixed at any point in space $(x_0\, y_0\, z_0)$, the normals to the three confocal surfaces of the second order passing through that point are made the new axes of the rectangular coordinates, ξ, η, ζ. The quantities k^2, k'^2, k''^2, are determined by the equations $k^2 = a^2 - a_0^2$, $k'^2 = a'^2 - a_0^2$, $k''^2 = a''^2 - a_0^2$, in which a^2, a'^2, a''^2, are the squares of the primary semi-axes of the three confocal surfaces ; f stands for the quantity

$$\frac{x_0^2}{a_0^2} + \frac{y_0^2}{b_0^2} + \frac{z_0^2}{c_0^2};$$

and ξ_0, η_0, ζ_0, are the coordinates of the centre of the surface.

It has also been observed by the same geometer, that

$$\frac{\xi^2}{k^2} + \frac{\eta^2}{k'^2} + \frac{\zeta^2}{k''^2} = 0$$

is the equation of the cone whose vertex is at the point $(x_0\, y_0\, z_0)$, and which envelopes the surface $(a_0\, b_0\, c_0)$; whilst

$$\frac{\xi_0\,\xi}{k^2} + \frac{\eta_0\,\eta}{k'^2} + \frac{\zeta_0\,\zeta}{k''^2} = 1$$

is the equation of the plane of contact of the cone and surface.

From this form of the equation of the surface of the second order we are enabled to deduce a general theorem ; the consideration of a particular case of which suggests a simple proof of Joachimsthal's theorem concerning geodetic lines, P D = *Const.*

If a perpendicular P *be let fall from the centre of a surface of the second order* $(a_0\, b_0\, c_0)$ *upon any tangent plane to a cone enveloping the surface, we shall have*

$$\frac{1}{P^2 L^2} = \frac{\operatorname{Cos}^2 \alpha}{(a^2 - a_0^2)^2} + \frac{\operatorname{Cos}^2 \beta}{(a'^2 - a_0^2)^2} + \frac{\operatorname{Cos}^2 \gamma}{(a''^2 - a_0^2)^2};$$

where L *is the length of the side of contact ;* α, β, γ, *are the angles it makes with the axes of the cone ; and* a, a', a'' *are*

the primary semi-axes of the three surfaces confocal to $(a_0 b_0 c_0)$, *which intersect at its vertex.*

This theorem may be proved as follows: The equation of the tangent plane to the cone is

$$\frac{\xi'\xi}{k^2} + \frac{\eta'\eta}{k'^2} + \frac{\zeta'\zeta}{k''^2} = 0,$$

ξ', η', ζ', being the co-ordinates of the point on the surface $(a_0 b_0 c_0)$, touched by the plane; and the square of the perpendicular P let fall from the centre on it is given by the formula,

$$P^2 = \frac{\left\{\frac{\xi'\xi_0}{k^2} + \frac{\eta'\eta_0}{k'^2} + \frac{\zeta'\zeta_0}{k''^2}\right\}^2}{\frac{\xi'^2}{k^4} + \frac{\eta'^2}{k'^4} + \frac{\zeta'^2}{k''^4}}$$

But, since the point $(\xi'\ \eta'\ \zeta')$ lies in the plane of contact, the numerator in this expression is equal to unity. Therefore, if we put

$$\xi' = L\cos\alpha, \quad \eta' = L\cos\beta, \quad \zeta' = L\cos\gamma,$$

we shall obtain the result stated above.

The quantity PL is evidently the same for the four sides of the cone L, L', L'', L''', whose directive angles are respectively (α, β, γ), $(\alpha, \pi - \beta, \gamma)$, $(\alpha, \pi - \beta, \pi - \gamma)$, $(\alpha, \beta, \pi - \gamma)$; and, if we denote by D the semidiameter of the surface parallel to L, the quantity PD will likewise be the same for them all; since the sides of the cone are proportional to the parallel semidiameters. Again, the planes of L and L'', L' and L''', pass through the internal axis of the cone; whilst those of L and L', L'' and L''', L' and L'', L''' and L, pass through its external axis.

Let us now suppose the vertex of the cone to approach indefinitely near to a point V on the surface: its internal axis becomes the normal; and the external axes ultimately coincide in direction with the tangents to the two lines of curvature passing through the point V. L and L'' may now be regarded as two successive elements of a geodetic line, since their plane

contains the normal. And, as it has been proved already that the quantity PD is the same for both of them, we are now in possession of a proof that, in passing from one element to the adjacent one, along a geodetic line traced on a central surface of the second order, the quantity PD remains unaltered. Let us next inquire what becomes of L and L' in the extreme case under consideration. These lines may be regarded as ultimately the elements of two lines making equal angles with the lines of curvature passing through V. We have, therefore, the theorem that if two right lines touch a surface of the second order at the same point, making there equal angles with a line of curvature passing through it, the quantity PD is the same for both. And, as a particular case, we have the theorem that, in passing from one element to the adjacent one, along a line of curvature traced on a central surface of the second order, the quantity PD remains unaltered.

Returning now to the case in which the vertex of the cone is supposed to be at a finite distance from the surface, we see that the four sides, L, L', L'', L''', are tangents to geodetic lines for which the quantity PD is the same, and which, therefore, touch the same line of curvature; and conversely, if two rectilinear tangents to the same geodetic line, or to geodetic lines for which the quantity PD is the same, intersect each other, they make equal angles with the axes of the cone which, from this point of intersection, envelopes the surface.

When the enveloping cone becomes a right one, we have PL or PD the same for all its sides; the geodetic lines to which they are tangents must, therefore, all converge to the same umbilic.

From what has been already stated, it is easy to deduce the following theorems:

If a closed flexible and inextensible cord be kept stretched by a style, being partly in free contact with a surface of the second order along a geodetic line, and partly free in space,

the style at the point of intersection of its straight portions will trace out another geodetic line upon a confocal surface.

And, if it be kept stretched, being partly in constrained contact with the surface along a line of curvature, and partly free in space, the style will trace another line of curvature upon a confocal surface.

June 26, 1848.

REV. HUMPHREY LLOYD, D. D., President,
In the Chair.

Sir W. R. Hamilton stated the following additional theorems respecting certain reciprocal surfaces, to which his own methods have conducted him.

If a plane quadrilateral $ABCD$ be inscribed in a given sphere, so that its four sides may be constantly parallel to four given straight lines; and if E, F be the two points of meeting of the two pairs of opposite sides, namely; E the meeting of the sides AB, CD, and F the meeting of BC, DA (prolonged if necessary); then the locus of the point E will be one ellipsoid, and the locus of the point F will be another ellipsoid reciprocal thereto.

And other pairs of reciprocal surfaces of the second degree may be generated in like manner, by changing the sphere to other surfaces of revolution of the second degree.

For instance, a pair of reciprocal cones of the second degree may be generated as the loci of two points E, F, which are, in like manner, the points of meeting of the opposite sides of a plane quadrilateral $ABCD$, inscribed in a circular section of a right-angled cone of revolution, with their directions in like manner constant. And a pair of reciprocal hyperboloids (whether of one or of two sheets) may, in like manner, be generated from an equilateral hyperboloid of revolution (of one or of two sheets).